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Sidelobe Reduction and Resolution Enhancement by Random Perturbations in Periodic Antenna Arrays

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ABSTRACT

A perturbational procedure for reducing sidelobe level and increasing resolution of antenna arrays with uniform amplitude excitation by introducing random element displacement with respect to the periodic configuration is presented. The element displacement is considered as a random variable subject to a certain probability distribution function (pdf). The simulated beampattern of the random array is obtained by ensemble averaging of many realizations of element displacement. For comparison, the beampattern is evaluated by using uniform and Gaussian pdfs. In the case of Gaussian distribution, it is found that the peak sidelobe can reach very low levels with increasing the number of elements, whereas it is almost fixed around – 30 dB in the case of perturbation by uniform distribution function. Moreover, as the number of elements increases, Gaussian distribution shows better beampattern with less number of sidelobes compared to the uniform distribution. This result is consistent with random array theory. In either distribution, and for a fixed number of elements, lower sidelobe levels and less beamwidths are obtained, in contrast with amplitude tapering techniques.

Keywords: *Aperiodic array*, *Gaussian distribution*, *periodic array*, *random array*, *uniform distribution*.

I. INTRODUCTION

Uniform linear arrays have a predictable beampattern with recognized mainlobe width and sidelobe structure where the first (peak) sidelobe is always at about – 13 dB [1]. This limitation has motivated the development of randomly spaced (aperiodic) arrays [2, 3]. The inherent randomness of the antenna array distribution alleviates typical half-wavelength spacing requirement for grating-lobe free scanning, this eliminates the presence of grating lobes in the array beampattern, allowing for potentially alias-free high-resolution patterns at a lower cost (using fewer elements) than standard periodic designs [4] over wide bandwidths [5]. This provides a general framework to relax physical constraints on element sizes and array spacing, which can in turn be applied to sparse arrays [6] of electrically larger radiators with broadband or wideband capabilities. The increased average spacing in these cases can reduce the effects of mutual coupling and scan blindness [7]. Randomly distributed arrays have shown a notable reduction in sidelobe level using fewer elements without amplitude tapering [3] thus alleviating the need for complicated feed system and the increase in main beamwidth.

In [8] acoustic scattering by an ensemble of scatterers whose positions are given random perturbations compared to a periodic arrangement is considered in one dimension. The perturbed periodic scattering medium is the effective medium averaged over 10^4 –10^5 realizations of randomness. Harrington [9] used a similar approach to reduce the sidelobe level but the element displacements from periodic spacing were evaluated deterministically. It is shown that the sidelobes can be reduced in height to approximately $2/N$ times the mainlobe level, where N is the number of elements, with the same beamwidth as for the periodic array.

In this paper, the method of [8] is adopted to introduce random perturbations into periodic array element locations. The element displacement is taken as random variable according to a given probability density function (pdf). The random array beampattern is evaluated by averaging many realizations of random element displacement and compared between uniform and Gaussian pdfs. In either pdf, it is found that the peak sidelobe level is reduced substantially with higher resolution (less beamwidth) compared to that of conventional periodic array without using any amplitude tapering.

II. BASIC THEORY

Consider a linear array of N equally excited isotropic elements as shown in Fig. 1. If the elements are spaced periodically by half-wavelength and the first element is placed at origin, the location of the nth element will be nd, where n = 0, 1,…, N–1, and d = 0.5 (*all dimensions are presented in wavelength*). Assuming that the element locations are shifted by independent and identically distributed random variables ε_n in accordance to a common pdf $p(\varepsilon_n)$. In this case, the element positions of the perturbed periodic (random) array are

$$x_n = (n + \varepsilon_n)d, \qquad n = 0,\dots, N-1 \tag{1}$$

It should be noted that the locations of the first and last elements in the array are unchanged to fix the aperture length of the array for a given number of elements. Thus, $\varepsilon_n = 0$ for $n = 0$ and $n = N-1$. The corresponding array factor will be

$$F(\theta|x_n) = \frac{1}{N}\sum_{n=1}^{N} e^{j2\pi x_n \sin\theta} \tag{2}$$

where θ is the observation angle as shown in Fig. 1. In this paper, the simulated array factor is averaged on a number of realizations of ε_n. This number of realizations is increased until the results could not be better anymore (in terms of the beamwidth and peak sidelobe level), i.e., a state of saturation is reached. It was found that after 10000 realizations of ε_n, no further improvement in results is obtained. Consequently, all simulation results in this paper are taken as the ensemble average of 10000 realizations of ε_n.

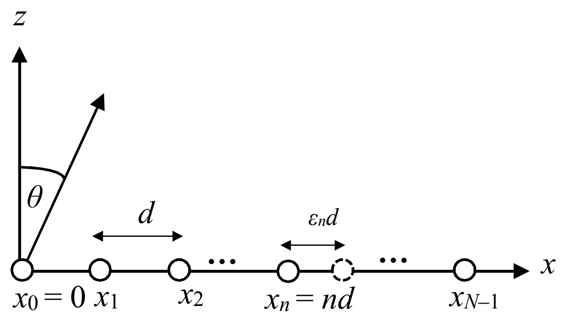

Fig. 1: The geometry of an N-element linear array with the periodic position of the nth element, $x_n = nd$, is randomly perturbed to a new position, $x_n = (n + \varepsilon_n)d$, where d is the periodic interelement spacing.

Two types of probability distribution functions $p(\varepsilon_n)$ are taken, namely uniform and Gaussian pdfs. In the case of uniform distribution function, ε_n lies in the interval $[-a, a]$. Thus, the uniform pdf is given by

$$p_u(\varepsilon_n) = \frac{1}{2a}, \qquad |\varepsilon_n| \le a \tag{3}$$

While in Gaussian distribution function, ε_n has zero mean and variance σ^2. In this case, the Gaussian pdf becomes

$$p_G(\varepsilon_n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{{\varepsilon_n}^2}{2\sigma^2}}, \qquad -\infty < \varepsilon_n < \infty \tag{4}$$

From (1), it can be inferred that the nth element location is only dependent on ε_n for a given N. Therefore, x_n has the same distribution as ε_n, i.e.,

$$p(x_n) \sim p(\varepsilon_n) \tag{5}$$

According to the central limit theory of large numbers, the beampattern of a random array converges to its expected (mean) one as $N \to \infty$ [3]. In fact, this expected beampattern is the Fourier transform of the pdf of element location [10]. This can be shown by taking the mathematical expectation of (2) as follows:

$$E_x\{F(\theta|x)\} = NE_x\left\{\frac{1}{N} e^{j2\pi x \sin\theta}\right\} = E_x\left\{e^{j2\pi x \sin\theta}\right\} = \int_{-\infty}^{\infty} p(x) e^{j2\pi x \sin\theta}\, dx = F_0(\theta) \tag{6}$$

It can be indicated from (6) that $F_0(\theta)$ and $p(x)$ are a Fourier transform pair. In other words, the pdf of element location plays the role of current density with respect to the expected pattern, which can be matched to any required radiation pattern derivable from a real current density through a proper selection of the pdf of element location [11].

III. Beampattern Characteristics of Randomly Perturbed Linear Array

A. Deciding a and σ

To determine the half-interval a of the uniform pdf as well as standard deviation σ of the Gaussian pdf, a linear array of N = 12, 50, and 100 elements is simulated and the variation of peak sidelobe level with both a and σ is evaluated. In the case of uniform distribution, it is found that the peak sidelobe has a minimum at about a = 8, 33, 67 for N = 12, 50, and 100, respectively, as shown in Fig. 2. Hence, a can be taken nearly as

$$a = 2N/3 \tag{7}$$

In the case of Gaussian pdf, it is found that the peak sidelobe level has a minimum at about σ = 5.5, 24.5, and 49.5 for N = 12, 50, and 100, respectively, as shown in Fig. 3, which is the aperture length of the array in each case, i.e., σ can nearly be taken as

$$\sigma = (N-1)/2 \tag{8}$$

Inspection of Figs. 2 and 3 reveals that the peak sidelobe level has more significant variation with a in the case of uniform pdf than it has with σ in the case of Gaussian pdf. In other words, in the case of Gaussian distribution, the peak sidelobe level has a little dependence on σ particularly after the optimum σ, the one has the minimum peak sidelobe level, after which the peak sidelobe level has very little variation with increasing σ as shown in Fig. 3. This can be inferred to the lower variance of the Gaussian spatial distribution as compared to the uniform distribution, and consequently its more stable beampattern (i.e., more stable peak sidelobe level) [14] .Also, from Figs. 2 and 3, it can be seen that increasing N in uniform distribution has a little effect on minimum peak sidelobe level (it is almost fixed around – 30 dB), while increasing N in Gaussian distribution leads to decreasing in minimum peak sidelobe level (see Sections III-B and -C for more details).

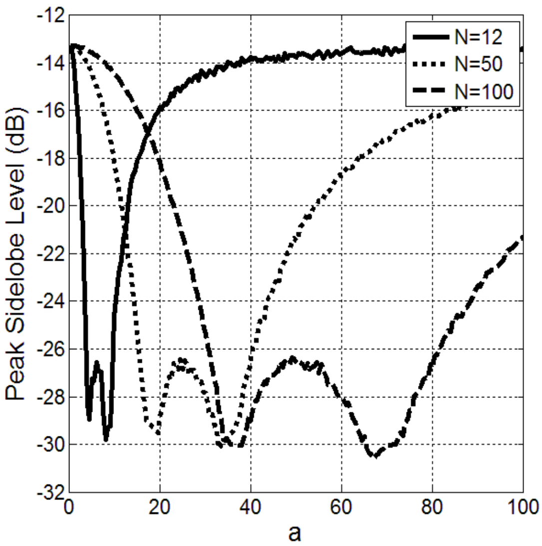

Fig. 2: Dependence of peak sidelobe level on a for a uniformly perturbed N-element linear array.

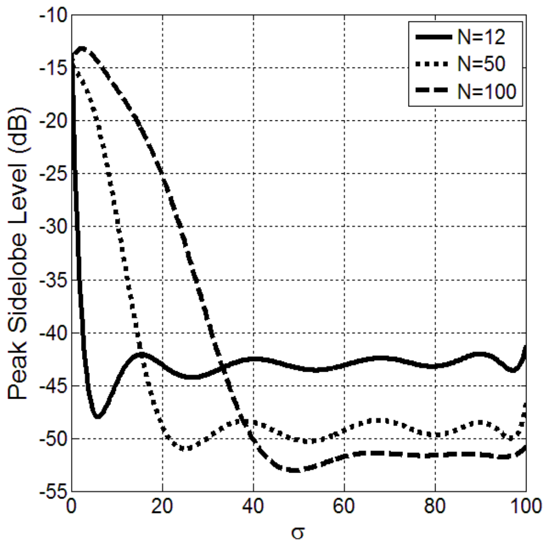

Fig. 3: Dependence of peak sidelobe level on σ for a Gaussian perturbed N-element linear array.

B. Array Factor

The 12-element linear array is simulated with a and σ as in (7) and (8), respectively. The final element positions of uniformly and Gaussian perturbed periodic arrays along with the unperturbed periodic array (with the nominal $\lambda/2$ spacing) are shown in Fig. 4. The corresponding array factors of the three types of array are shown in Fig. 5. It can be seen that the peak sidelobe level of uniformly perturbed array is about – 30 dB. The peak sidelobe level of the Gaussian perturbed array is about – 47 dB. Whereas the peak sidelobe level of the unperturbed periodic array is at the typical – 13 dB level. Moreover, as shown in Fig. 5, the periodic array has the widest mainbeam and the mainbeam in the case of uniform perturbations is slightly wider than in the case of Gaussian perturbations.

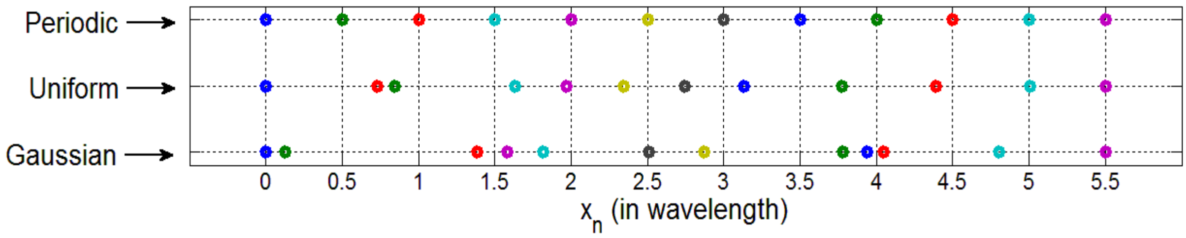

Fig. 4: The element positions of periodic, uniformly perturbed, and Gaussian perturbed 12-element linear array.

To show the effect of N on behavior of the pattern for both uniformly and Gaussian perturbed arrays, the number of elements is increased to be 100 as shown in Fig. 6. It can be noted that while the peak sidelobe of the uniformly perturbed array is still at about – 30 dB, the corresponding one of Gaussian perturbed array has decayed to a level of about – 53 dB. Moreover, it can be noted that the mainlobe of the uniform perturbations case is still a little wider than in the case of Gaussian perturbations. Furthermore, it is obvious that beampattern of the uniformly perturbed array has a larger number of near-in sidelobes than that of the Gaussian perturbed array. This property makes the perturbed array with Gaussian distribution function of a better beampattern with less overall number of sidelobes.

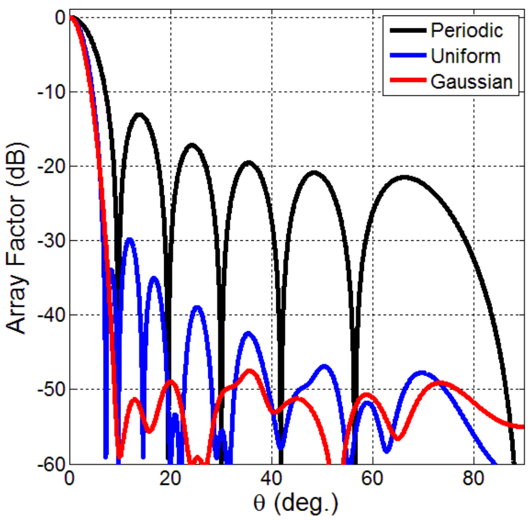

Fig. 5. The array factor corresponding to the element positions of Fig. 4.

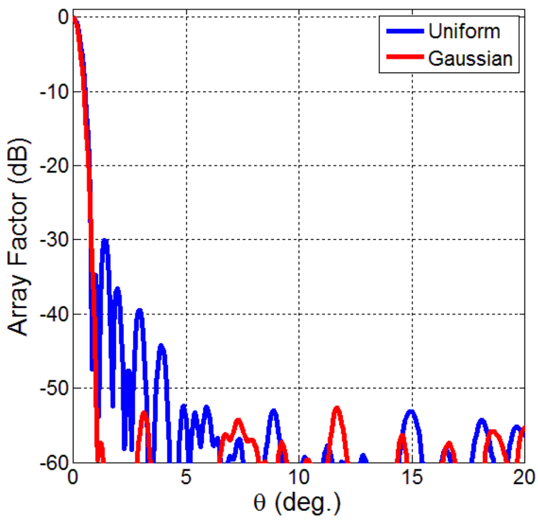

Fig. 6. The array factor for both uniformly and Gaussian perturbed 100-element linear array.

The behavior of randomly perturbed arrays is inspected for a large N by simulating 1000-element linear array as shown in Fig. 7. It can be noted that as $N \rightarrow \infty$ the beampattern converges asymptotically to the expected beampattern of the random array, and this expected beampattern is actually the Fourier transform of the pdf of element location (see (6)). This can be evidently seen through the close-up of the mainbeam in Fig. 7, where the uniform pdf has a beampattern of an oscillatory *sinc* function, the Fourier transform of a uniform (rectangular) function, while the Gaussian distribution transforms into a Gaussian beam (another Gaussian function).

In the case of uniform perturbations, and as mentioned above, as N increases, the beampattern approaches a *sinc* function that rapidly decreases with increasing the spatial parameter θ. As shown in Fig. 7, the peak sidelobe level of uniformly perturbed array is still around – 30 dB for $N = 1000$. Since the peak sidelobe level does not depend on the aperture length and is less sensitive to the value of N [12], high peaking sidelobes can only be eliminated by increasing the aperture length (here, increasing N means increasing aperture length), leaving the majority of sidelobe maxima concentrated relatively close to the mainlobe as shown in the close-up of Fig. 7. This is also a logical statement complimenting realism such that sidelobes are nearly unworkable to remove from the mainlobe region in practice [13].

Also as mentioned above, when N increases, the Gaussian beampattern converges to a Gaussian beam that contains neither nulls nor sidelobes as shown in Fig. 7. Moreover, the beampattern decays exponentially with a rate proportional to the variance σ^2 [14]–[16] which is also in a good agreement with Fig. 3. The Gaussian beampattern is similar to the beampattern for uniformly perturbed array. However, in the latter case, the oscillatory *sinc* function results in nulls and sidelobes in the beampattern. Meaning that when the radiation intensity is free of sidelobes, it does not have the same opportunity of having high peaks in the beampattern like the patterns with oscillatory functions. This can be clearly seen through Figs. 5–7.

C. Peak Sidelobe Level

Figure 8 shows the peak sidelobe level when N increases from 100 to 1000 for periodic, uniformly perturbed, and Gaussian perturbed linear arrays. As expected, the peak sidelobe level of unperturbed periodic array is almost constant at –13 dB. For uniformly perturbed array, the peak sidelobe is also almost fixed and slightly less than – 30 dB with high peaking sidelobes remain concentrated around the mainbeam (Figs. 5–7). This is an artifact of a uniform excitation, and similar to the concentration of significant sidelobes around the mainbeam in large periodic arrays. This is in contrast to the Gaussian perturbed array where the peak sidelobe is consistently decreasing to very low levels as N increases (it is less – 50 dB for arrays have more than 100 elements and less than – 60 dB for array sizes more than 700 elements). This means that as $N \to \infty$, the beampattern approaches zero in the sidelobe region. Therefore, the sidelobes with high peaks are less probable in the case of Gaussian spatial distribution.

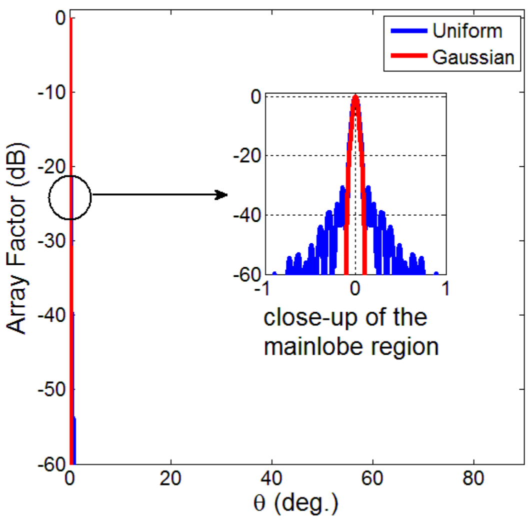

Fig. 7: The array factor with a close-up of the mainlobe region for both uniformly and Gaussian perturbed 1000-element linear array.

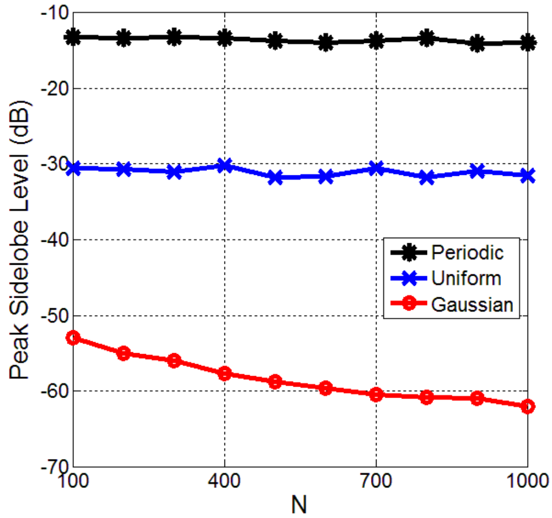

Fig. 8: The peak sidelobe level as a function of N for periodic, uniformly perturbed, and Gaussian perturbed linear array.

D. 3-dB Beamwidth

The 3-dB beamwidth is plotted in Fig. 9 against *N* for the periodic and the randomly perturbed arrays. As it can be seen, the beamwidth of periodic array is, in general, larger than that of randomly perturbed one (whether it is uniformly or Gaussian perturbed). The difference in 3-dB beamwidth between them decreases as *N* increases until they are equal at $N = 700$, after that the periodic array has constant 3-dB beamwidth at 0.1°, whereas the 3-dB beamwidth of the randomly perturbed array goes down to a very low value that approaches zero resulting in a very sharp mainbeam. For the randomly perturbed arrays, the mainbeam of the uniformly perturbed is slightly wider than that of the Gaussian perturbed until $N = 200$ after which they almost have the same 3-dB beamwidth.

In general, the 3-dB beamwidth is inversely proportional to the aperture length of the array [13] (here, it equals $(N - 1)/2$, so increasing *N* means an increase in aperture length). This infers a sparse arrangement of radiators provides, on average, a very narrow beam.

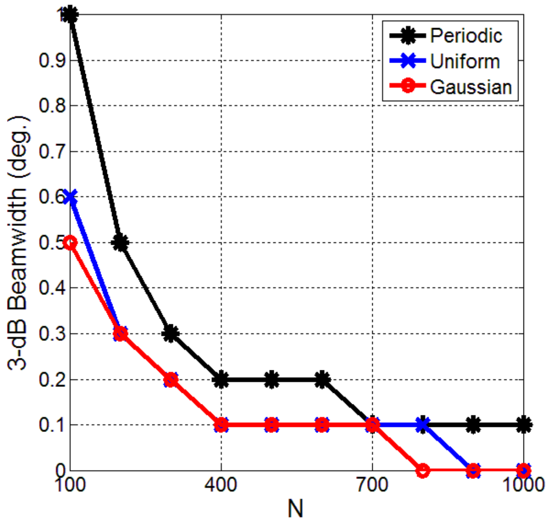

Fig. 9: 3-dB beamwidth as a function of *N* for periodic, uniformly perturbed, and Gaussian perturbed linear array.

IV. Conclusion

In this paper, the sidelobe level of a linear periodic array is reduced, besides enhancing its resolution by giving its elements random displacements from their periodic positions according to some pdf. Two pdfs are considered namely, uniform and Gaussian. While their resolutions are almost the same, and still better than that of the periodic array, the smooth decay of Gaussian pdf promises lower sidelobes in the beampattern than those of the uniform pdf. Since the array elements maybe become very close to each other when they are randomly displaced, the nearest-neighbor constraint [17] can be imposed to overcome this problem. Moreover, the approach of this paper can be easily extended to planar antenna arrays. It is shown that randomly perturbed array, especially with the Gaussian pdf, is getting better with low sidelobes and pencil mainbeam as *N* becomes larger. Therefore, the proposed approach seems promising when applied to large arrays.

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